Contact with nonequilibrium

Pattern formation in nonequilibrium media

Stabilization of nonequilibrium phases

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Brief history of time

The stages of nonequilibrium physics:

1. Dynamical characterization of the equilibrium condition;

   Reversibility of Detailed balance

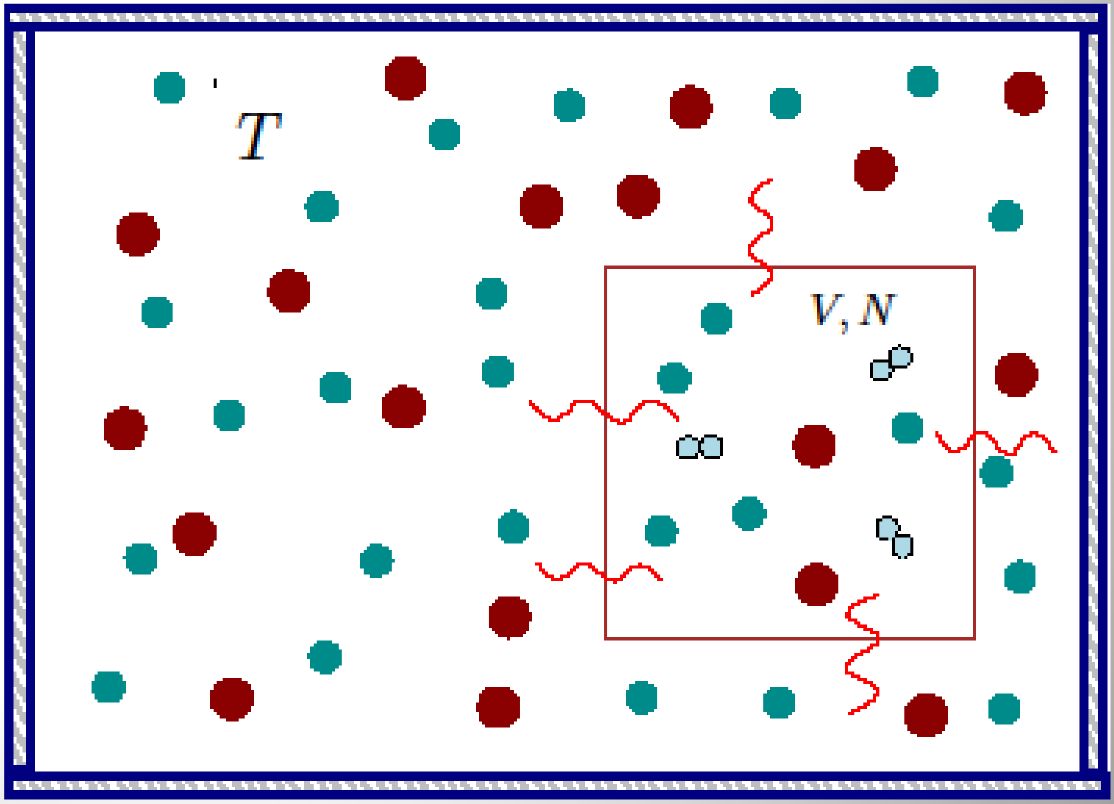
2. Description of systems that are open to spatio-temporally separated but different equilibrium reservoirs;

   Local detailed balance

3. Study of systems in contact with nonequilibrium media.

   What is induced dynamics?

System in contact with equilibrium:
relaxation along free energy landscape,
Free energy = thermodynamic potential for
statistical force



Pressure, osmosis,… as thermodynamic forces

Static fluctuations in equilibrium:

Connects with work and forces via the miraculous relation,

Force on X is

$$-\int \mathrm{d}\eta \, \frac{1}{Z(X)} e^{-\beta H(\eta,X)} \, \nabla_X H(\eta, X) = -\nabla_X \mathcal{F}(X)$$

for free energy acting *truly* as

thermodynamic potential,

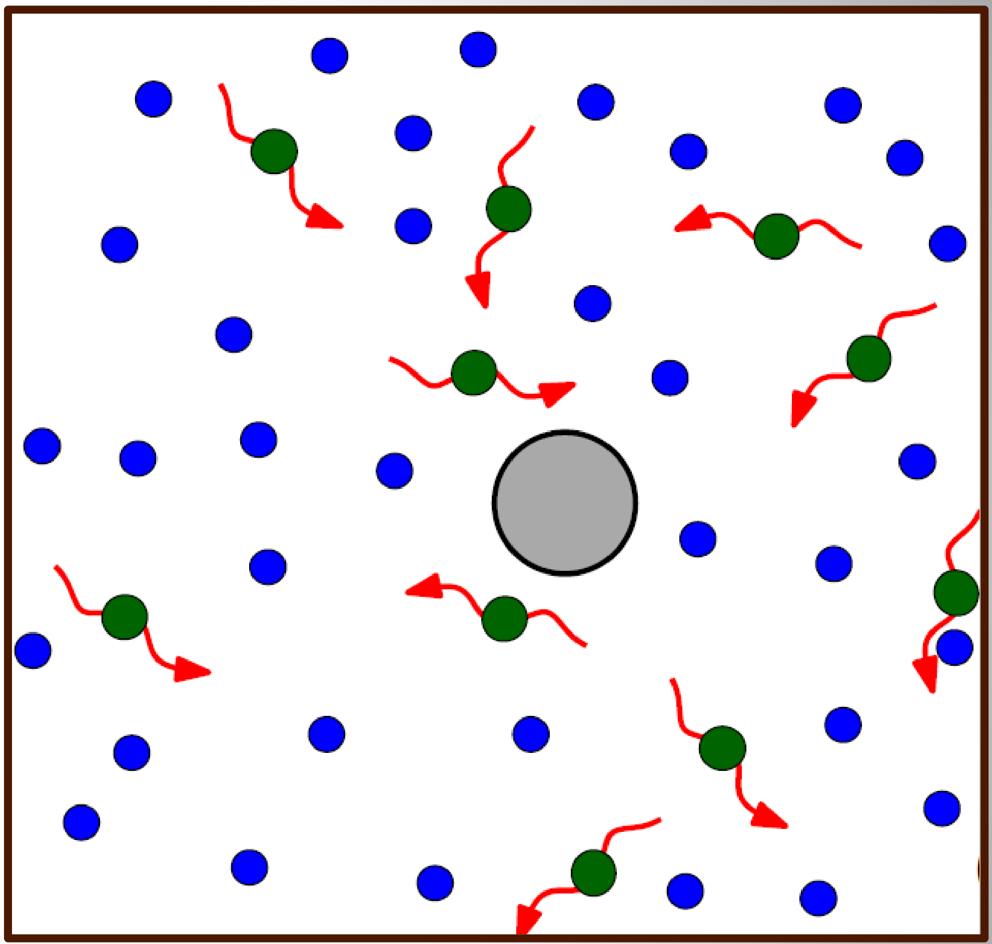
$$\mathcal{F}(X) = -k_B T \log Z(X)$$

Relaxation to equilibrium reduced to GRADIENT FLOW

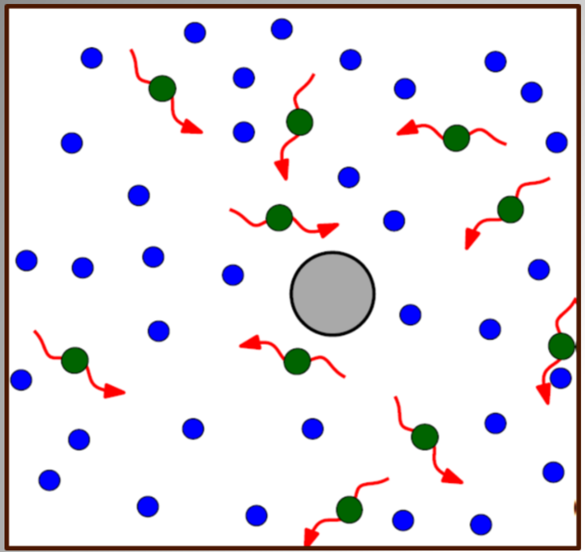
Variational evolution: patterns stored as minima of potential

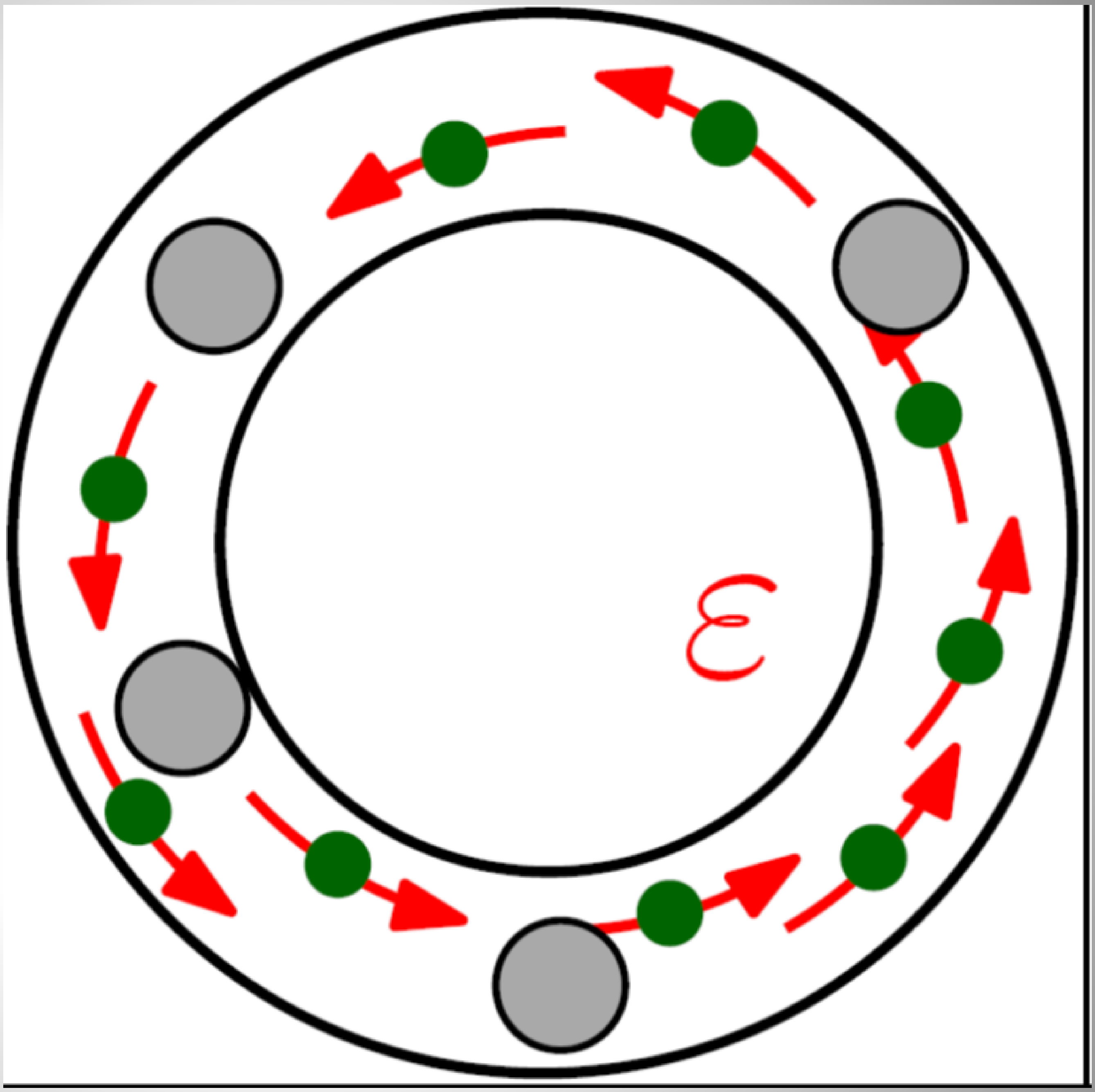
Set-up

Probe interacting with **nonequilibrium medium** connected to equilibrium reservoir(s)



Time-scale separation: probe is slow, medium relaxes fast to stationary state




ε

Langevin-Smoluchowski dynamics

Force = systematic part + friction + noise

? Systematic force = nature of induced force

? 2nd fluctuation-dissipation theorem in friction/noise relation

$$M'\ddot{Q} + \int_0^{+\infty} \mathrm{d}\tau\, K'_{\gamma'}(\tau)\dot{Q}_{t-\tau} = F_{\mathrm{tot}}(Q_t) + \zeta_t$$

? Nature of noise

for a probe in a medium, colloid, Brownian motion,...

What is new in nonequilibrium?

Systematic force can obtain nongradient part, can delete free energy contribution, need not be additive, need not satisfy action-reaction,..

Fluctuation-dissipation relation typically violated because of frenetic contribution in response (in friction).

Can get nonGaussian noise because of long tails and long range correlations in nonequilibrium bath.

What is new in nonequilibrium?

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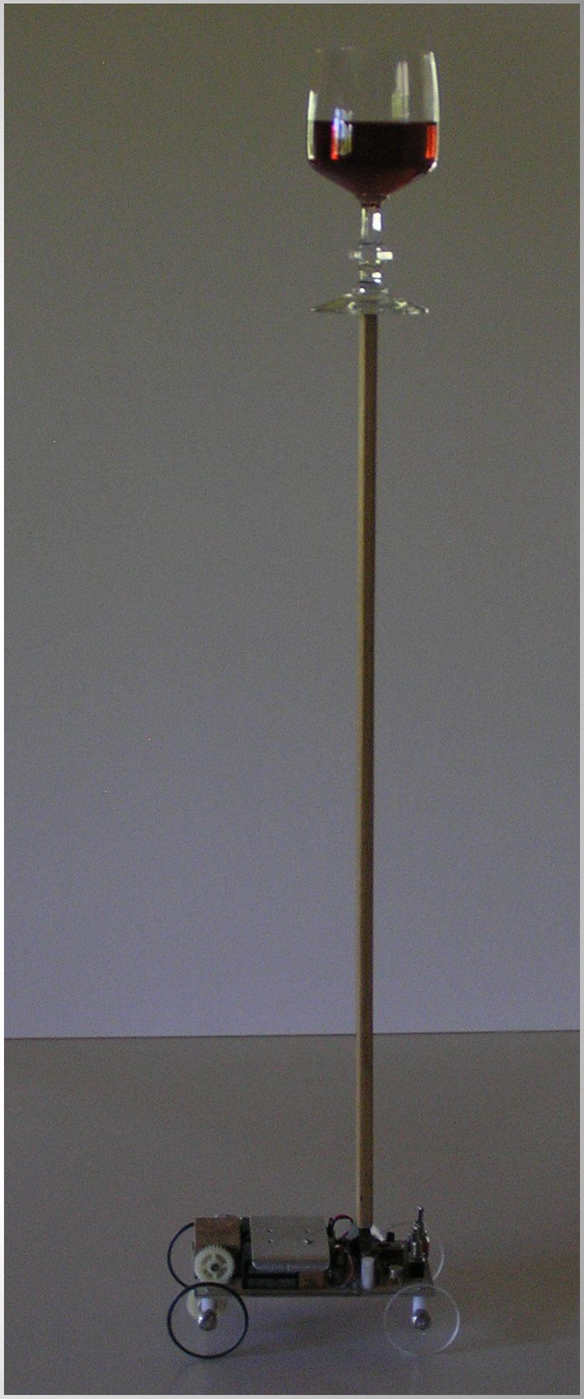
Illustration: how nonequilibrium can stabilize fixed points.

Stabilizing the metastable or even the unstable…

Standard examples:

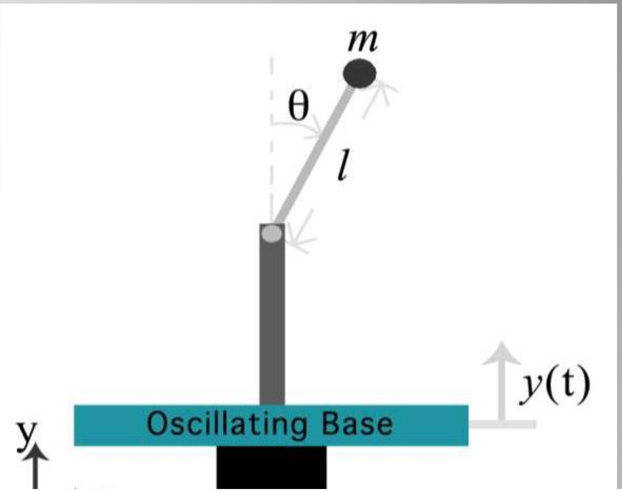
-Via feedback, dynamical control



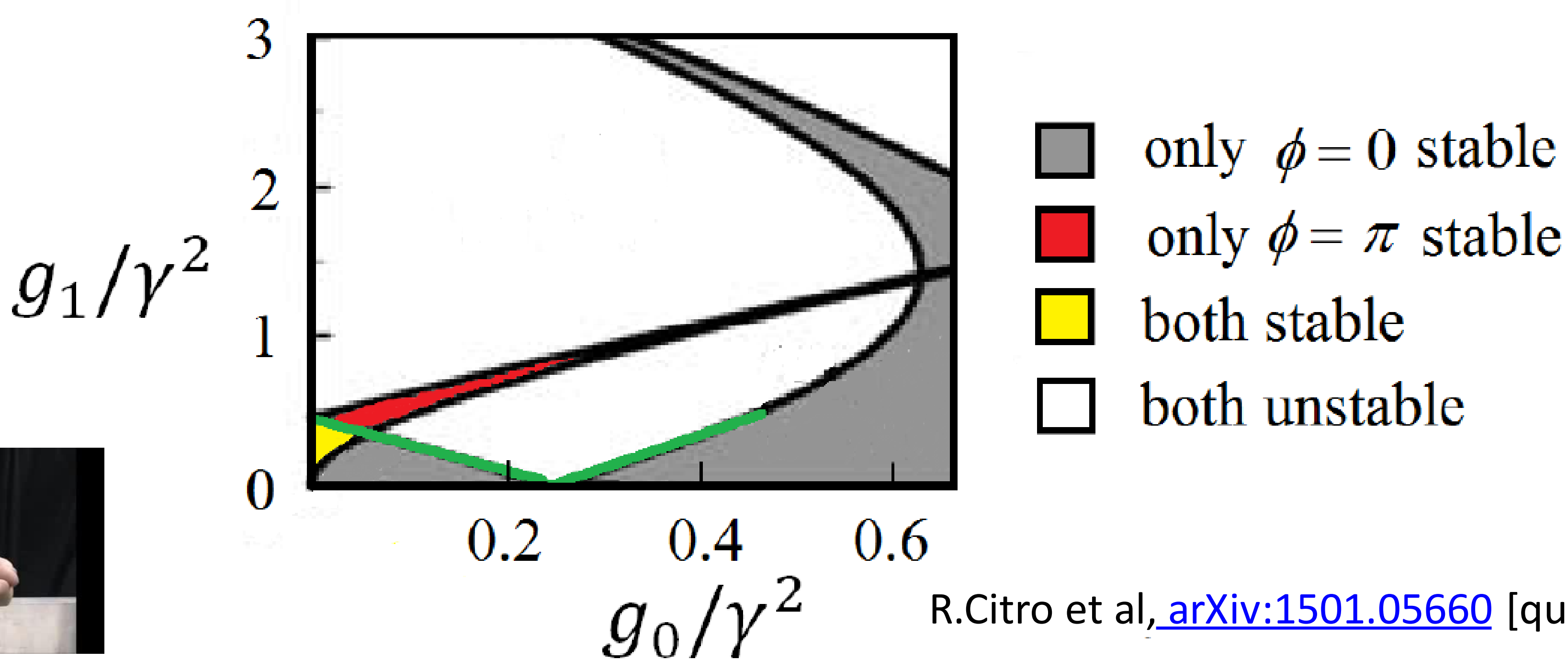
Stabilizing the metastable or even the unstable…

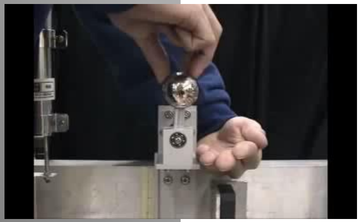
Much less trivial:

Stephenson-Kapitza (inverted) pendulum



$$H(t) = \frac{1}{2}p^2 - g(t)\cos(\phi), \quad \text{with} \quad g(t) = g_0 + g_1\cos(\gamma t)$$





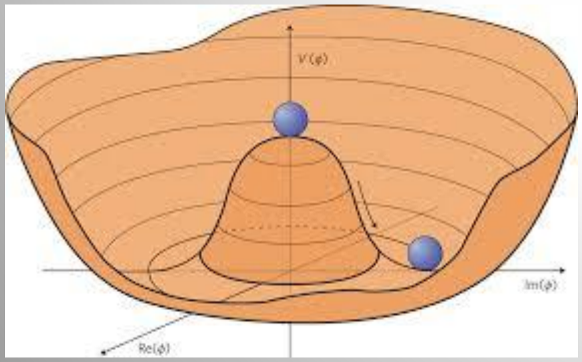
R.Citro et al, arXiv:1501.05660 [quant-ph]

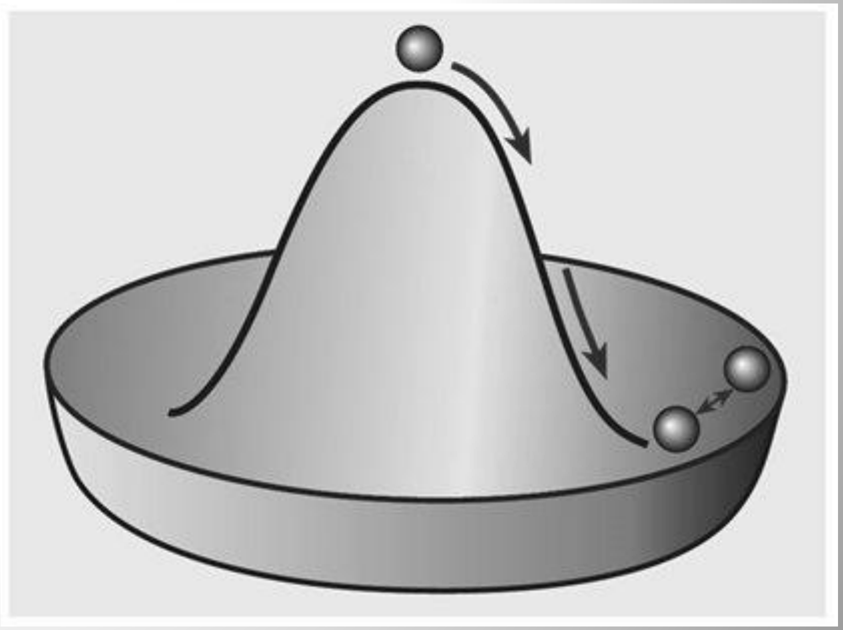
Stabilizing the metastable or even the unstable…

Consider many particles undergoing a Mexican-shape self-potential (2 dim).

Short range attraction to probe

Origin is unstable fixed point for probe





$$\dot{y}_t = F(y_t) - \nabla U_x(y_t) + (2T)^{1/2}\xi_t\,, \qquad \nabla\cdot F = 0$$

We take the potential and driving field

$$U_x(y) = V(|y|) + U_I(|y-x|), \qquad F(y) = \varepsilon |y| \omega(|y|)\,\hat{e}_\varphi$$

$$V(r) = \begin{cases} k_0 e^{-\frac{r^2}{2\sigma_0^2}} & \text{for } 0 \le r \le R \\ 0 & \text{for } r > R \end{cases}$$

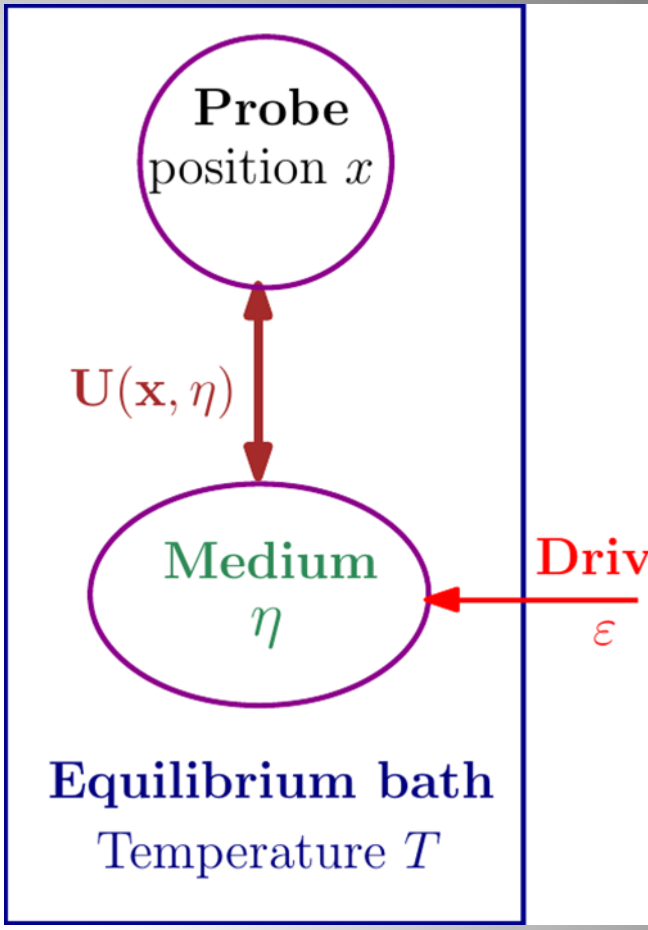
$$U_I(r) = -\lambda e^{-\frac{r^2}{2\sigma^2}}$$

$$V(r) = k_0 e^{-\frac{r^2}{2\sigma_0^2}} + k_w e^{r-\sigma_w}$$

or

$$U_I(x,y) = -\lambda \left[1 - \frac{(x-y)^2}{\sigma^2}\right]^2$$

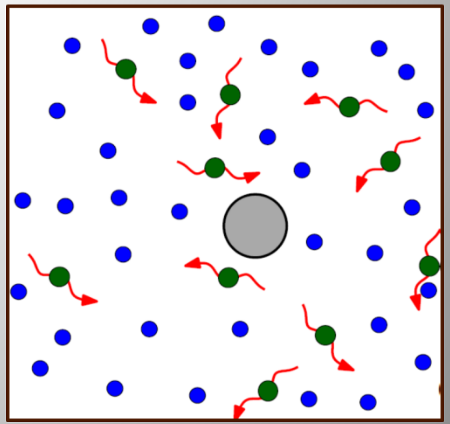
Statistical force



Probe (x) coupled to nonequilibrium medium through energy U(x,η)

Systematic force:

$$f(x) = -\int \rho_x(d\eta)\nabla_x U(x,\eta) = -\langle \nabla_x U(x,\eta)\rangle^x.$$



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1)Effective dynamics of probe?

-New (un)stable phases?

-Additivity of forces?

-New phenomenology – oscillatory motion

-Pressure? Equation of state in active media?

2) Properties of medium?

-Static fluctuations?

-Excess dynamical activity?

-Excess thermodynamic quantities?

Shown: stability of origin as fixed point of probe increases with rotation amplitude of medium.

E.g. plot of effective spring constant m, second order effect

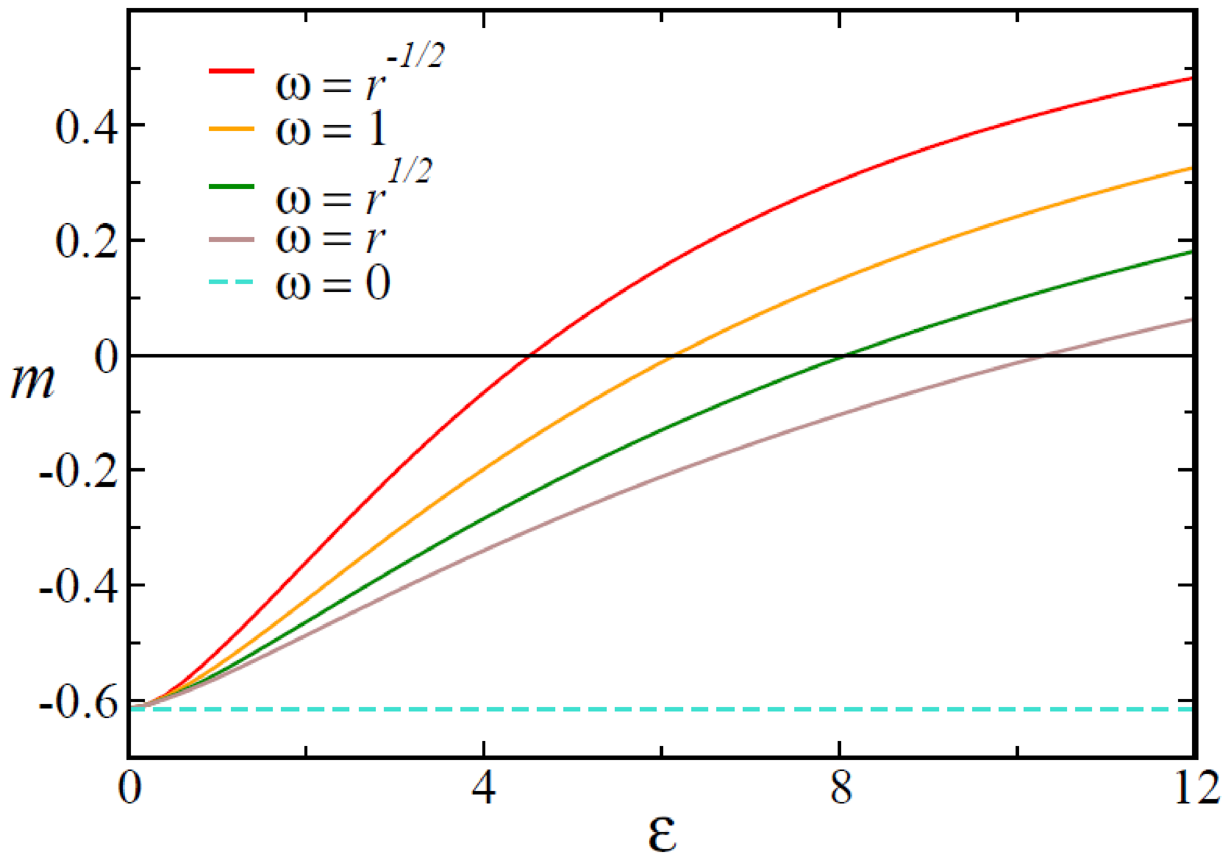
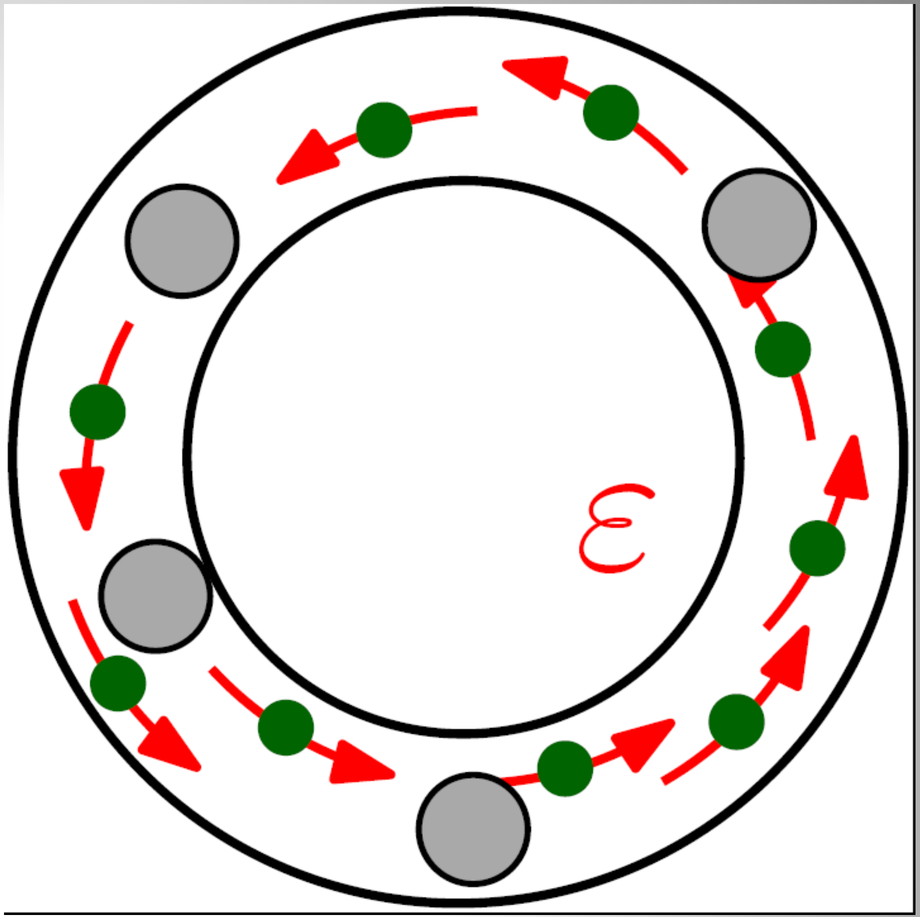


Fig. 1: Stiffness for various rotation profiles as function of the driving ε. Parameters: $T = 1$, $\lambda = 1$, $k_0 = 3/4$, $\sigma = 1/2$, $\sigma_0 = 1$, and $R = 5$.

Multiple probes in short-range interaction with driven medium



medium

$$\gamma \frac{\mathrm{d}\eta_t}{\mathrm{d}t} = \varepsilon - \frac{\partial U(x,\eta)}{\partial \eta} + \left(\frac{2\gamma}{\beta}\right)^{1/2} \xi_t$$

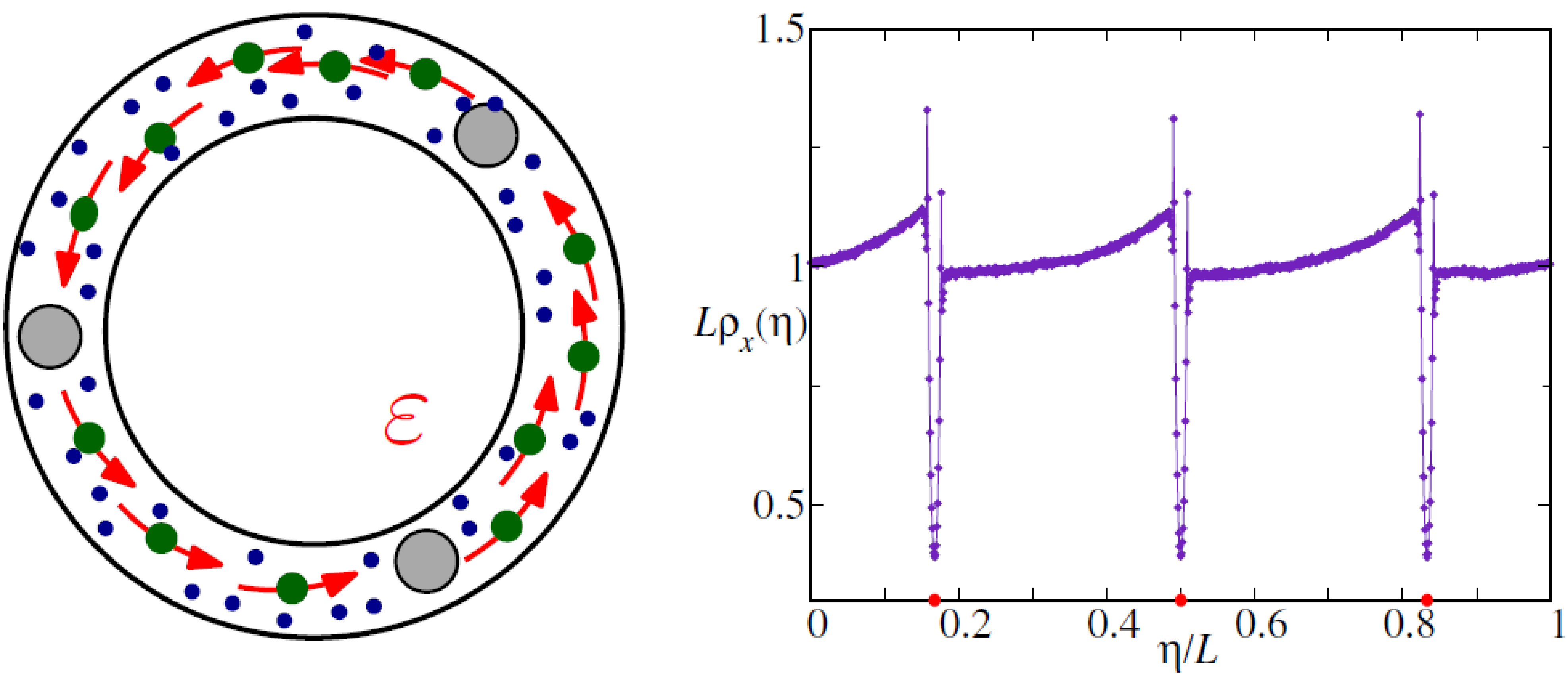
coupling

$$u(z) = \begin{cases} u_0 \left[1 - \left(\frac{z}{\delta}\right)^2\right]^2 & \text{for } |z| \le \delta \\ 0 & \text{otherwise} \end{cases}$$

Statistical force on αth-probe

$$f_\alpha(x) = -\oint \frac{\partial U(x,\eta)}{\partial x_\alpha} \rho_x(\eta)\,\mathrm{d}\eta = -\oint u'(x_\alpha - \eta)\,\rho_x(\eta)\,\mathrm{d}\eta$$

Colloidal density profile for given probe positions in case or repulsion



$$f_\alpha(x) = f^{\mathrm{drift}}(x) + f_\alpha^{\mathrm{int}}(x)$$

$$f^{\mathrm{drift}}(x) = \zeta A\, j_x$$

$$f_\alpha^{\mathrm{int}}(x) = -\frac{\zeta j_x B}{\ell_d} \sum_{\gamma \neq \alpha} e^{-(x_\gamma - x_\alpha)^+/\ell_d}$$

Stability of equidistant "crystal" configuration

$$x^*_\alpha(t) = v^* t + \frac{L}{N}\alpha \quad (\alpha \bmod N)$$

$$\Gamma \dot{y}_\alpha = \sum_\gamma M_{\alpha\gamma} y_\gamma\,, \qquad M_{\alpha\gamma} = \frac{\partial f_\alpha(x^*)}{\partial x_\gamma}$$

$$x_\alpha = x^*_\alpha + y_\alpha$$

$$M_{\alpha\alpha} = -\sum\nolimits_{\gamma\neq\alpha} M_{\alpha\gamma} \qquad M_{\alpha\gamma} = m_{\gamma-\alpha}$$

$$m_\alpha = \frac{\zeta j^* B}{\ell_d^2\,(1 - e^{-L/\ell_d})}\, e^{-\frac{L}{N\ell_d}\alpha}\,, \qquad \alpha = 1, \ldots, N-1$$

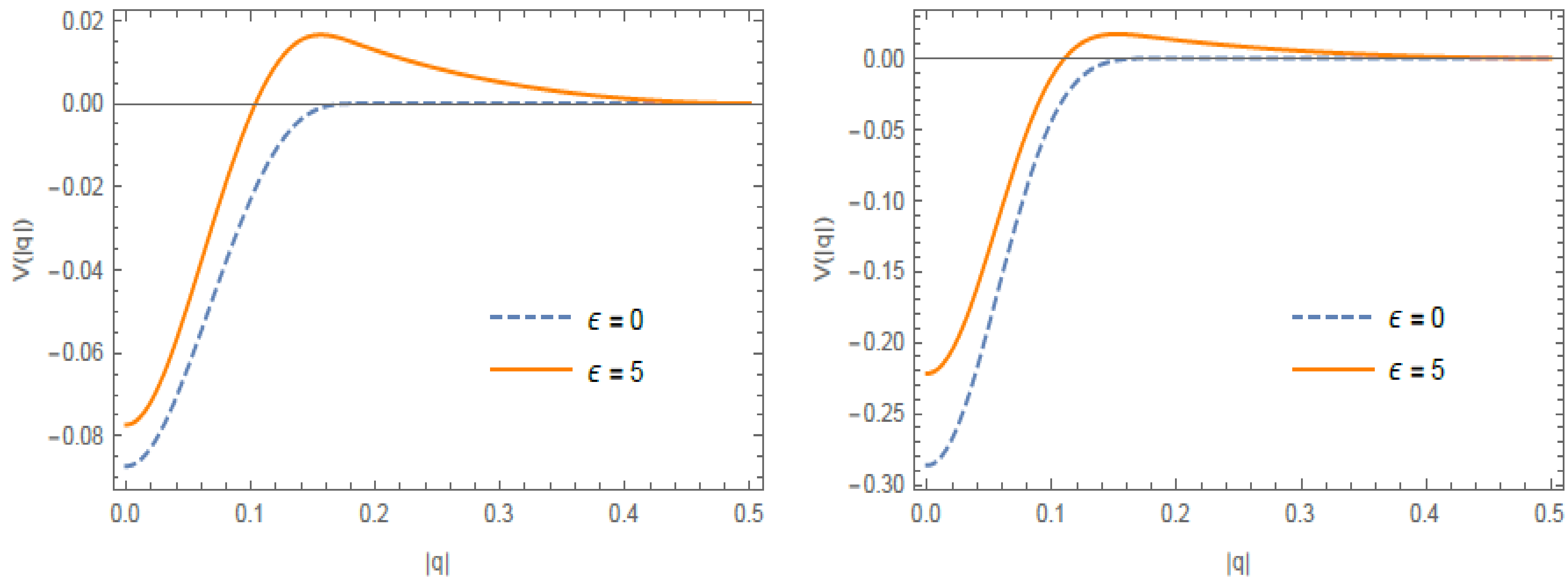


FIG. 1: Induced probe–probe interaction potential for (a) repulsive ($u_0 = 1$) and (b) attractive ($u_0 = -1$) probe–medium coupling. The other parameters are $\varepsilon = 5$, $\delta = 0.1$, $\beta = 1$, $\gamma = 1$, $\rho^o = 1$, and $L = 1$.

NEW ATTRACTOR: stable EQUIDISTANT CONFIGURATION OF PROBES

FLOW TOWARDS *CRYSTAL* FORMATION

References

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